a possibility and of the arising advantages, let us consider the reconstruction of the previous scattering scenario where both the imaginary and the real part of the contrast function are unknown. By considering a “bare” single frequency approach \( CG \rightarrow SF \), \( U^{CG \rightarrow SF} \approx 3250 \) are the unknowns parameters and the mean time per iteration is of about \( \Delta t = 5.80 \text{s} \). Dealing with a multifrequency implementation, the dimension of the unknowns space becomes \( U^{CG \rightarrow MF} \approx 2960 \) and \( U^{CG \rightarrow MF} \approx 45.64 \text{s} \).

On the contrary, an exploitation of the proposed IMSA-MF strategy would require a mean time per iteration of \( t_{\text{ IMSA-MF}} \approx 11.14 \text{s} \) for reconstructing \( U^{IMS A \rightarrow MF} \approx 6250 \) parameters (such a configuration provides a resolution level of the same order of the \( CG \rightarrow MF \)) without leading to an increasing of the computational costs (with respect to the standard single-step approach), thanks to the adaptive allocation at each step of the unknowns.

**IV. CONCLUSION**

In this paper, starting from a set of representative test cases, a comparative assessment of two multiresolution approaches exploiting multifrequency data has been carried out. The objective was that of giving some indications on the most suitable strategy able to improve in a non-negligible fashion the reconstruction accuracy with a reasonable amount of computational resources compared to that of single-step methodologies. As a matter of fact, unlike the IMSA-FH, the IMSA-MF provided numerical proofs of the enhanced effectiveness in processing multifrequency information. Moreover, the numerical and experimental analysis pointed out current limitations of the proposed approach in reconstructing lossy profiles and the computational needs of the three-dimensional multifrequency multiscaling procedure that, even though demanding, is considerably more effective than the single step procedure. Eventually, great care should be exercised in choosing the illumination frequencies and in determining the informative amount of multifrequency scattering data. As a matter of fact, such issues strongly affect the performance of each multiresolution multifrequency procedure since they play a key-role in allowing a good tradeoff between suitable resolution level in the reconstruction, occurrence of local minima, and ill-conditioning of the whole inversion.

**REFERENCES**


**Mobile Terminal Location for MIMO Communication Systems**

Ji Li, Jean Conan, and Samuel Pierre

**Abstract**—We propose a novel approach in the context of multiple-input multiple-output communication systems to determine the position of mobile terminals based on estimated multipath signal parameters such as angle-of-arrival, angle-of-departure and delay-of-arrival using only one Base Station. This approach minimizes the errors occurring from the estimation of multipath parameters and gives the position of the mobile terminal by simultaneously resolving a set of algebraic location equations. The root-mean-square (RMS) errors are measured and compared with the Cramer-Rao Lower Bound to demonstrate the performance of the proposed method.

**Index Terms**—Least squares (LS), mobile location, multipath, multiple-input multiple-output (MIMO), Taylor series.

**I. INTRODUCTION**

Mobile terminal location will be one of the most exciting features of the next generation wireless systems [1]. The conventional methods of position-location (PL) systems estimate the position of a mobile source by measuring the parameters such as angle-of-arrival (AOA), time-of-arrival (TOA), time-difference-of-arrival (TDOA) of the signal from the source [2].

A promising approach to improve the performance of mobile location systems is to use antenna arrays in both transmitter and receiver sides. Such multiple-input multiple-output (MIMO) communication systems could exploit the spatial properties of the multipath channel. It is then possible to estimate channel parameters such as AOA, angle of departure (AOD), and delay of arrival (DOA) simultaneously in a multipath environment by using adaptive array signal processing techniques [3], [4].

**REFERENCES**


In this paper, based on estimated multipath signal parameters in MIMO communication systems, we propose a novel approach to determine the position of mobile stations using only one Base Station. This approach intends to minimize the error occurring from the estimation of multiple paths and gives the position of mobile terminal by simultaneously calculating a set of nonlinear algebraic position equations. To our knowledge, no similar results have been proposed in the published literature.

The remainder of this work is outlined as follows. In Section II, we establish the system model for mobile location in MIMO communication systems. Section III proposes a least squares (LS) solution with Taylor series linearization for the hybrid TDOA/AOA/AOD location method. The Cramer-Rao lower bound (CRLB) is also derived. The performance of the proposed method is evaluated via computer simulation in Section IV. Conclusions are given in Section V.

II. MODEL FOR POSITION-LOCATION OF MOBILE TERMINAL

In this work, we consider a simplified 3GPP MIMO channel model [5] with $N$ resolvable propagation paths between the transmitter and the receiver sites and where each scatterer has single reflected signal. We assume that the multipath signal parameters such as AOD, AOA, and DOA have been measured using the method outlined in [3] and [4] with respect to a common bearing direction. As illustrated in Fig. 1, let $(x_b, y_b), (x_m, y_m)$, and $(x_i, y_i)$ denote the true position of, respectively, the base station (BS), the mobile station (MS) and the $i$th scatterer. The values of $(x_m, y_m)$ and $(x_i, y_i)$ are known and must be estimated. Let $r_i'$, $r_i''$ be, respectively, the lengths of the segments forming the $i$th path. Finally, let $\theta_i$, $\phi_i$ be the angles of departure and arrival for the $i$th path from the mobile terminal to the base station. We assume that $\theta_i$ and $\phi_i$ are from $-\pi/2$ to $\pi/2$.

If we have a line-of-sight path available, the position of mobile station can be calculated easily using simple geometry knowledge. However, in practice, the LOS is not always available or can not be distinguished easily. Moreover, the measurements of DOA, AOA, and AOD always contain errors due to the hostile wireless propagation environment.

From Fig. 1, it is straightforward to obtain $\theta_i$ and $\phi_i$ as a function of positions of the mobile terminal and the scatterers

$$\begin{align*}
\theta_i(x_m, y_m, x_i, y_i) &= \arctan\left(\frac{y_i - y_m}{x_i - x_m}\right) \\
\phi_i(x_m, y_m, x_i, y_i) &= \arctan\left(\frac{y_i - y_m}{x_i - x_b}\right)
\end{align*}$$

(1)

for $i = 1, \ldots, N$. Similarly, the TDOA can be computed

$$\tau_i(x_m, y_m, x_i, y_i) = \frac{(r_i - r_1)}{c}, \quad i = 2, \ldots, N$$

(2)

where $r_i = r_i' + r_i''$ with

$$\begin{align*}
r_i' &= \sqrt{(x_i - x_m)^2 + (y_i - y_m)^2} \\
r_i'' &= \sqrt{(x_i - x_b)^2 + (y_i - y_b)^2}.
\end{align*}$$

(3)

The objective is to determine the unknown position $(x_m, y_m)$ from the exact position $(x_b, y_b)$ and uncertain measurements of $\theta_i$, $\phi_i$ and $\tau_i$.

Statistically, the measurements contain errors $n_{\theta_i}, n_{\phi_i}, n_{\tau_i}$ such that

$$\begin{align*}
\hat{\tau}_i &= \tau_i(x_m, y_m, x_i, y_i) + n_{\tau_i} \\
\hat{\theta}_i &= \theta_i(x_m, y_m, x_i, y_i) + n_{\theta_i} \\
\hat{\phi}_i &= \phi_i(x_m, y_m, x_i, y_i) + n_{\phi_i}
\end{align*}$$

(4)

where $i = 1, \ldots, N$ for $\hat{\theta}_i$ and $\hat{\phi}_i$, $i = 2, \ldots, N$ for $\hat{\tau}_i$.

For this method, it is not necessary to have a LOS path available. With $N$ multipath available, we have $N$ AOA measurements, $N$ AOD measurements, and $N - 1$ TDOA measurements. Consequently when the number of paths $N \geq 4$, we have $(3N - 1)$ measurements and $(2N + 2)$ unknown parameters, so the system is over-determined yielding a nonlinear estimation problem which can be solved by a least-squares method.

III. HYBRID TDOA/AOA/AOD LOCATION METHOD

A. Least Squares (LS) Method With Taylor Series Linearization

In this section, we derive an iterative LS location estimator to solve the nonlinear TDOA/AOA/AOD equations for the MS location. Let the vector $X = [x_m, y_m, x_1, y_1, \ldots, x_N, y_N]^T$ represent the true position of the mobile station and the scatterers. Then define $F$ as the $(3N - 1)$ column vector

$$F(X) = \begin{bmatrix}
\tau_1(x_m, y_m, x_1, y_1) \\
\vdots \\
\tau_i(x_m, y_m, x_i, y_i) \\
\vdots \\
\phi_1(x_m, y_m, x_1, y_1)
\end{bmatrix}.$$ 

(5)

The estimation model in (4) for the unknown $(2N + 2)$ vector $X$ in the presence of additive Gaussian noise can be written in matrix form

$$M = F(X) + \mathbf{n}$$

(6)

where the vector $M = [\hat{\tau}_1, \hat{\theta}_1, \ldots, \hat{\phi}_i]^T$ represents the $(3N - 1)$ measurement values [8]. The measurement errors $\mathbf{n} = [n_{\tau_1}, n_{\theta_1}, \ldots, n_{\phi_i}]^T$ is a multivariate random vector with a $(3N - 1) \times (3N - 1)$ covariance matrix of the form

$$Q = \begin{pmatrix}
Q_{\tau} & 0 & 0 \\
0 & Q_{\theta} & 0 \\
0 & 0 & Q_{\phi}
\end{pmatrix}$$

(7)

where $Q_{\tau}$ is the covariance matrix for the TDOA measurement errors, $Q_{\theta}$ and $Q_{\phi}$ are the covariance matrices for the AOD and AOA measurement errors respectively.
If $\mathbf{X}$ is regarded as an unknown but non-random vector, and $\mathbf{n}$ is assumed to have zero mean and Gaussian distribution, the maximum likelihood (ML) estimation problem for $\mathbf{X}$ is given by

$$\hat{\mathbf{X}} = \arg \min_{\mathbf{X}} [\mathbf{M} - \mathbf{F}(\mathbf{X})]^T \mathbf{Q}^{-1} [\mathbf{M} - \mathbf{F}(\mathbf{X})]. \quad (8)$$

If $\mathbf{F}(\mathbf{X}) = \mathbf{H} \mathbf{X}$ is a linear function with $\mathbf{H}$ a constant matrix, the ML estimator gives a LS solution

$$\hat{\mathbf{X}} = (\mathbf{H}^T \mathbf{Q}^{-1} \mathbf{H})^{-1} \mathbf{H}^T \mathbf{Q}^{-1} \mathbf{M}. \quad (9)$$

In order to determine a reasonably simple estimator, the nonlinear function $\mathbf{F}(\mathbf{X})$ has to be linearized. The most straightforward linearization method is to use the Taylor series expansion. Provided the vector $\delta \mathbf{X}$ is small, we can write the following approximation:

$$\mathbf{F}(\mathbf{X} + \delta \mathbf{X}) \approx \mathbf{F}(\mathbf{X}) + \mathbf{G} \delta \mathbf{X}. \quad (10)$$

The gradient $\mathbf{G}$ of $\mathbf{F}$ with respect to $\mathbf{X}$ is defined as the $(3N - 1) \times (2N + 2)$ matrix whose $i$th row is the gradient of the scalar $F_i$ with respect to $\mathbf{X}$

$$\mathbf{G} = \frac{\partial \mathbf{F}}{\partial \mathbf{X}} = \begin{pmatrix} \frac{\partial F_1(\mathbf{X})}{\partial x_1} & \cdots & \frac{\partial F_1(\mathbf{X})}{\partial X_{2N+2}} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_{3N-1}(\mathbf{X})}{\partial x_1} & \cdots & \frac{\partial F_{3N-1}(\mathbf{X})}{\partial X_{2N+2}} \end{pmatrix}. \quad (11)$$

If $\mathbf{X}^{(n-1)}$ is an approximation of $\mathbf{X}$ and $\delta \mathbf{X}$ represents a small perturbation, then from (10)

$$\mathbf{F}(\mathbf{X}^{(n-1)} + \delta \mathbf{X}) \approx \mathbf{F}(\mathbf{X}^{(n-1)}) + \mathbf{G} \delta \mathbf{X}. \quad (12)$$

Reporting this relation in (8), the ML estimator $\hat{\delta \mathbf{X}}^{(n-1)}$ for $\delta \mathbf{X}$ is then given by (10) with $\mathbf{M}$ replaced by $\mathbf{F}(\mathbf{X}^{(n-1)})$ and $\mathbf{G}$ used in place of $\mathbf{H}$. Assuming $\mathbf{G}^T \mathbf{Q}^{-1} \mathbf{G}$ is nonsingular, the LS estimate of the linearized estimate is

$$\delta \mathbf{X}^{(n-1)} = (\mathbf{G}^T \mathbf{Q}^{-1} \mathbf{G})^{-1} \mathbf{G}^T \mathbf{Q}^{-1} [\mathbf{M} - \mathbf{F}(\mathbf{X}^{(n-1)})]$$

providing a new estimator $\mathbf{X}^{(n)} = \mathbf{X}^{(n-1)} + \delta \mathbf{X}^{(n-1)}$.

The procedure can be applied iteratively starting from an initial guess $\mathbf{X}^{(0)}$ assumed to be close to $\mathbf{X}$ (i.e., under the small measurement errors assumption) to yield a sequence $\mathbf{X}^{(n)}$ of estimates. This process is repeated until the value of $\delta \mathbf{X}$ becomes smaller than a desired threshold, indicating convergence [6].

### B. Cramer-Rao Lower Bound

The CRLB provides a lower bound on the variance of any unbiased parameter estimators. Hence it is of interest to compare the proposed estimator with the optimum. The CRLB for our estimation problem is given by

$$\Phi = \left( \frac{\partial F^T(\mathbf{X})}{\partial \mathbf{X}} \mathbf{Q}^{-1} \frac{\partial F(\mathbf{X})}{\partial \mathbf{X}} \right)^{-1} \quad (14)$$

where $\frac{\partial F(\mathbf{X})}{\partial \mathbf{X}}$ is given by (11) evaluated at the true value of $\mathbf{X}$. $\Phi$ is obtained for all other elements. The AOD covariance matrix $\mathbf{Q}_d$ is used to solve the location equations with only three paths, which leads to a unique solution for MS and scatterers. Simulations show that at most five iterations are required for Taylor-series solutions to converge. A validity test is implemented after each iteration. We compute $\det(\mathbf{G}^T \mathbf{Q}^{-1} \mathbf{G})$ and reject the input data or the position guess if this number is too small. To detect the failure of convergence, we compute the trace of $(\mathbf{G}^T \mathbf{Q}^{-1} \mathbf{G})^{-1}$ at the end of each iteration, and after five steps or so, start to compare it with that of the previous step. If the ratio is not much less than unity, we assume the process is not converging.

Table I compares the RMS errors of the position location with the square root value of CRLB. The CRLB is obtained from the first two diagonal elements of (14). The TDOA noise standard deviation is set to be $\sigma^2_t = 100 m$, whereas the AOD and AOA noise standard deviation are $3$ degree and $1$ degree respectively. From the results, we can see that the MS position can be estimated with high accuracy and the RMS of the proposed method can fully take advantage of the information richness of MIMO systems due to multipath dispersion. We outline the following advantages due mostly to the fact that single base station is involved.

1) The time synchronization is straightforward, and furthermore, PL information collection by the network is facilitated; 2) Most PL systems require LOS path which do not always exit in practice. Our method does not need this requirement. In fact, it will even find the LOS path in that case.

### IV. Simulations and Results

The performance of the proposed mobile location method for MIMO communication systems has been investigated by computer simulations. Let the BS located at the center of the coordinate, and MS unknown position be $(3000, 3000)$ m. There are $N$ scatterers randomly distributed within a scatter circle with the center at the MS. The RMS location error of MS position are obtained from 10,000 independent runs.

For simplicity, we assume that the signals and noises are Gaussian random process. The TDOA covariance matrix $\mathbf{Q}_t$ used is similar to the one defined in [7] which has TDOA variance $\sigma^2_t$ for diagonal elements and $0.5\sigma^2_t$ for all other elements. The AOD covariance matrix $\mathbf{Q}_d$ and the AOA covariance matrix $\mathbf{Q}_a$ are diagonal with AOD variance $\sigma^2_a$ and AOA variance $\sigma^2_o$ respectively. The initial guess can be obtained by solving the location equations with only three paths, which leads to a unique solution for MS and scatterers. Simulations show that at most five iterations are required for Taylor-series solutions to converge. A validity test is implemented after each iteration. We compute $\det(\mathbf{G}^T \mathbf{Q}^{-1} \mathbf{G})$ and reject the input data or the position guess if this number is too small. To detect the failure of convergence, we compute the trace of $(\mathbf{G}^T \mathbf{Q}^{-1} \mathbf{G})^{-1}$ at the end of each iteration, and after five steps or so, start to compare it with that of the previous step. If the ratio is not much less than unity, we assume the process is not converging.

Table I compares the RMS errors of the position location with the square root value of CRLB. The CRLB is obtained from the first two diagonal elements of (14). The TDOA noise standard deviation is set to be $\sigma_t = 100 m$, whereas the AOD and AOA noise standard deviation are $3$ degree and $1$ degree respectively. From the results, we can see that the MS position can be estimated with high accuracy and the RMS of the proposed method approaches the square root value of CRLB very closely as the number of multipaths increases.

In Fig. 2, the cumulative distribution of squared errors are compared as the number of scatterers increases from 4 to 10. It is clear that the proposed algorithm performs better with added scatterers as additional information is provided to the estimation process.

In Fig. 3, we show a 3-D illustration of the RMS estimation with different TDOA and AOD noise measurement, whereas the AOA noise standard deviation is set to be 1 degree. The simulation results show that the maximum value of RMS is lower than 350 m with 7 scatterers.

The CRLB is derived in (14) for the proposed location method. In Fig. 4, we show a 3-D illustration of the square root value of CRLB.
Fig. 2. Square root error with different number of scatterers.

Fig. 3. RMS location error with different TDOA and AOD noise measurement.

with different TDOA and AOD noise measurement, whereas the AOA noise standard deviation is set to be 1 degree.

V. CONCLUSIONS

By using a set of measured multipath signal parameters, such as TDOA between pairs of paths, and AOD, AOA for each path, it is possible to estimate the position of the mobile terminal so as to minimize the effect of the measurement noise. To solve the nonlinear hybrid TDOA/AOA/AOD location equations, the proposed solution uses an iterative LS method combined with Taylor series linearization. Preliminary results on the potential of this technique have been provided. Further investigation is currently underway to determine the robustness of the method with respect to position location.

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